Improved data driven model free adaptive constrained control for a solid oxide fuel cell

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Abstract: Solid oxide fuel cells (SOFCs) are considered the most important in the field of fuel cells. However, SOFCs give a challenging control problem due to their slow dynamics, complex non-linearity, and operating constraints. In this study, an improved model free adaptive constrained control scheme is proposed to solve the SOFC control problem with I/O measurement data. Since the control terms appear within the unknown non-linear dynamic, an improved compact form dynamic linearisation data driven modelling method is used to simplify the SOFC system. In the design procedure of the controller, a novel dynamic anti-windup compensator is used to deal with the move magnitude and rate saturations of SOFC control input. Moreover, the authors also theoretically prove that all the signals in the closed-loop system are uniformly ultimately bounded based on Lyapunov stability analysis method. Finally, simulation results are given to demonstrate the effectiveness and potential of the proposed constrained control scheme.

1 Introduction

Solid oxide fuel cells (SOFCs) are a class of fuel cells characterised by the use of a solid oxide material as the electrolyte. SOFCs use a solid oxide electrolyte to conduct negative oxygen ions from the cathode to the anode. The electrochemical oxidation of the oxygen ions with hydrogen or carbon monoxide, thus occurs on the anode side. Therefore, a great deal of attention is paid to the use of SOFCs to generate electricity. Due to the SOFCs do not produce radiation or air pollution [1–3].

Mechanism modelling of SOFCs has been investigated and developed since the early 1990s, which are especially beneficial for designing and testing control strategies [4–8]. Achenbach [4, 5] investigated the transient cell voltage response to changes in temperature and current density. Padullés et al. [6] given a SOFC model based on species dynamics. However, due to the instability of the electrochemical reaction and different operation states in SOFCs, it is difficult to use a fixed mathematical model analysis and test control strategies of SOFCs. Over the last two decades, when the whole model information can not be accessed, there are a number of researches which show that it appears to be much effective for neural network (NN) and fuzzy logic techniques to identify complex non-linear systems [9–12]. Yet, the high convergence speed, the over-heating phenomenon and the avoidance of local minima still cannot be guaranteed for NN. Moreover, there is no common method to choose the number of the fuzzy rule base and the hidden units of common NN.

In order to prevent over used and under used fuel conditions, the desired range of fuel utilisation is from 0.7 to 0.9 [13]. In addition, control input limitations are always presented in real physical systems. If the controller is designed irrespective of these limitations, the later appearance of a saturation may cause undesired behaviour in the closed loop: it leads to performance degradation or even to instability [14]. At present, most control methods based on intelligent algorithm and model predictive control (MPC) are proposed for SOFCs [13, 15–20]. In [16], a Hammerstein model was adopted to describe the non-linear dynamic properties of the SOFC. The Hammerstein model was consisted of a radial basis function (RBF) NN in cascade with an autoregressive exogenous input model. A non-linear MPC was proposed to control the voltage based on an improved RBF network optimised with genetic algorithm (GA) in [17]. In [15], the authors developed a novel offset-free input to state stable fuzzy predictive controller via identified fuzzy model. In [13], the authors proposed a constrained MPC strategy to solve the SOFC control problem based on a support vector machine and GA. Obviously, we need adaptive updating parameters of the dynamics model for SOFCs due to the time-varying electrochemical reaction. As far as I know, the adaptive control method ia used for SOFC power control system [21, 22] in a few papers, but the control saturation problem and fuel utilisation of SOFC have not been paid enough attention.

In this paper, we focus on how to design a data driven MFAC based on Lyapunov method for a SOFC with move control input saturation. On the basis of previous works [26, 27], we present an improved CFDL modelling method and an adaptive constrained control scheme with I/O measurement data. Due to the magnitude and rate saturations of control input and fuel utilisation constraint are existing in the SOFC, based on the online identified improved CFDL model, a novel dynamic anti-windup compensator
is introduced to deal with the saturation problem. Moreover, built on the Lyapunov method, the stability of the closed-loop system is analysed for the proposed control method. Simulation results are given to demonstrate the effectiveness and potential of the proposed constrained control scheme. The major originalities of this dissertation are displayed as follows:

i. An improved CFDL data driven modeling is given for SOFC.
ii. A novel dynamic anti-windup compensator is proposed to deal with the move control input saturation.

The rest of this paper is structured as follows. In Section 2, a brief description of the SOFC is given. In Section 3, main results of constrained controller design are given. Simulation results are presented to show the effectiveness of the proposed technique in Section 4. Finally, some conclusions are made at the end of this paper.

2 Problem formulation for SOFC

The working principle of SOFCs is the same as that of other fuel cells: is equivalent to the ‘inverse’ device of water electrolysis in principle (see Fig. 1). The single cell consists of an anode, a cathode and solid oxide electrolyte. The anode is the place for fuel oxidation, and the cathode is the place for oxidant reduction. The two poles, both contain a catalyst, accelerating the electrochemical reaction of the electrode. When working, it is the equivalent of a DC power supply, whose anode is the negative power supply and cathode is the positive power supply.

Continue to pass into the fuel gas on the anode side of SOFC, such as hydrogen (H\textsubscript{2}), methane (CH\textsubscript{4}), city gas (CO) and so on. The fuel gas is adsorbed on the surface of the catalytic anode to diffuse into the interface of the anode and the electrolyte through the porous structure of the anode. Continue to pass into oxygen or air on the cathode side. The cathode surface with a porous structure adsorbs oxygen. Due to the catalytic action of the cathode, O\textsubscript{2} turns into O\textsuperscript{2–} after obtaining electron. Under the action of the chemical potential, O\textsuperscript{2–} enters into the solid oxide ion conductor working as an electrolyte. Since the concentration gradient causes diffusion, that finally arrives at the interface of the solid electrolyte and the anode, reacting with fuel gas, and the lost electrons return to the cathode through the external circuit.

The anode reaction and cathode reaction of the SOFC are shown as follows

\begin{align*}
&\text{Anode reaction : } H_2 + O^{2-} \rightarrow H_2O + 2e^- \\
&\text{Cathode reaction : } O_2 + 4e^- \rightarrow 2O^{2-}
\end{align*}

In this paper, take the SOFC dynamical model widely accepted as the object of study \cite{3, 11}, which is shown in Fig. 2, where \( V_{dc} \) is the stack output voltage (V), \( q_f \) is the natural gas (e.g. H\textsubscript{2}) flow rate (mol/s), and \( I \) expresses the measurable external current load (A); \( p_{H_2}, p_{O_2}, \) and \( p_{H_2O} \) denote the partial pressures of hydrogen, oxygen, and water (Pa), respectively; and \( \eta_{ohmic} \) and \( \eta_{conc} \) are the input flow rates of hydrogen and oxygen (mol/s), respectively. Applying Nernst’s equation and taking into account ohmic, concentration, and activation losses (i.e. \( \eta_{ohmic}, \eta_{conc}, \) and \( \eta_{act} \)), the stack output voltage \( V_{dc} \) is described as follows \cite{15–18}

\[ V_{dc} = V_0 - \eta_{ohmic} - \eta_{conc} - \eta_{act} \]  

where

\[ V_0 = N_0 \left[ E_0 + \frac{R_0 T_0}{2F_0} \ln \left( \frac{p_{H_2}\sqrt{p_{O_2}/101,325}}{p_{H_2O}} \right) \right] \]  

\[ p_{H_2} = \frac{1}{K_{H_2}(1 + \tau_{H_2} s)} \left( \frac{1}{1 + \tau_f s} q_f - 2K_I I \right) \]  

\[ p_{O_2} = \frac{1}{K_{O_2}(1 + \tau_{O_2} s)} \left( \frac{1}{1 + \tau_f s} q_f - K_I I \right) \]  

\[ p_{H_2O} = \frac{2}{K_{H_2O}(1 + \tau_{H_2O} s)} K_I I \]  

\[ \eta_{ohmic} = \beta r, \quad \eta_{conc} = \partial + \beta \ln I \]  

\[ \eta_{act} = - \frac{R_0 T_0}{2F_0} \ln I \left( 1 - \frac{I}{I_C} \right) \]  

![Fig. 1 Schematic of the working principle of a SOFC](image1)

![Fig. 2 SOFC system dynamic model](image2)
The parameters of the SOFC system are summarised in Table 1 [13]. In the closed-loop control system of SOFC, there are main three aspects are considered, they are:

i. The dynamic model of SOFC shows the current load \( I \) \( = \) \( \frac{q}{2} \n - \n_{min} = \frac{\Delta_{1}}{\Delta_{1}} \n \), \( u_{\n} \) for \( \Delta_{1} \leq \Delta_{1} \n \leq \Delta_{1} \n \leq \Delta_{1} \n \). Hence, the designed controller must reduce the influence of current load \( I \) changes and keep the output voltage steady.

ii. From the stationary voltage–current characteristics of the used SOFC system, which are depicted in Fig. 3, it can be seen that the SOFC exhibits non-linear behaviour over a wide operating regime. Especially at low and high current loads, and an overloaded current leads to a rapid deterioration of the operating stack voltage, stronger non-linear characteristics of SOFC system will be obvious. So non-linear SOFC controller is better than linear SOFC controller.

iii. On the one hand, because the input \( q_{t} \) of the SOFC cannot change too fast within a small time interval due to the ‘inertia’ of the actuator. The control inputs are subject to magnitude and rate constraints as follows

\[
q_{t_{min}} \leq q_{t} \leq q_{t_{max}}, \quad \dot{q}_{t_{min}} \leq \dot{q}_{t} \leq \dot{q}_{t_{max}} \quad \tag{8}
\]

On the other hand, fuel utilisation of the SOFC system is one of the most important operating variables that can affect the performance of a SOFC, should also be kept within a safe range for as long as possible, which is defined as

\[
\rho = \frac{\dot{q}_{H_{2}} - \dot{q}_{O_{2}}}{\dot{q}_{H_{2}}^{0}} = \frac{\dot{q}_{H_{2}}^{0} - \dot{q}_{H_{2}}^{0}}{\dot{q}_{H_{2}}^{0} - \dot{q}_{H_{2}}^{0}} = 2K_{\rho}I \quad \tag{9}
\]

where \( \dot{q}_{H_{2}}^{0}, \dot{q}_{O_{2}}^{0} \) are the hydrogen input flow rate, reacted flow rate, and output flow rate, respectively. The desired range of fuel utilisation is from \( \rho_{min} = 0.7 \) to \( \rho_{max} = 0.9 \) [3–6]. To ensure a safe utilisation, the hydrogen flow should be within \( \left[ \dot{q}_{H_{2}}^{min}, \dot{q}_{H_{2}}^{max} \right] \) through the calculation of (9). Further, we can calculate the constraints \( \left[ \dot{q}_{H_{2}}^{min}, \dot{q}_{H_{2}}^{max} \right] \) on the fuel flow as

\[
\dot{q}_{H_{2}}^{min} = \max (\dot{q}_{H_{2}}^{min}, \dot{q}_{H_{2}}^{max}), \quad \dot{q}_{H_{2}}^{max} = \min (\dot{q}_{H_{2}}^{max}, \dot{q}_{H_{2}}^{min}) \quad \tag{10}
\]

Hence, the newly magnitude and rate constraints are redefined as

\[
\dot{q}_{H_{2}}^{min} \leq \dot{q}_{t} \leq \dot{q}_{H_{2}}^{max}, \quad \dot{q}_{H_{2}}^{min} \leq \dot{q}_{t} \leq \dot{q}_{H_{2}}^{max} \quad \tag{11}
\]

Remark 1: It seems that optimal control is the only systematic approach with an inherent capability of handling multiple constraints. The predictive feature of MPC provides a convenient mechanism to take into account multiple constraints such as input saturation, rate and higher order constraints and even state constraints [28, 29]. However, analysis of non-linear optimal control with multiple constraints remains a challenge.

The \( q_{t}, V_{dc}, \) and \( I \) are defined as the control input \( u \), control objective \( y \), and disturbance \( d \), respectively. The input–output relation of SOFC can be rewritten in the following discrete-time unknown SISO non-linear AutoRegressive with eXogenous input (NARX) model

\[
y(k + 1) = f (y(k), \ldots, y(k - n_{y}), u(k), \ldots, u(k - n_{u}), d(k), \ldots, d(k - n_{d})) \quad \tag{12}
\]

where \( n_{y}, n_{u}, \) and \( n_{d} \) are the unknown orders. Obviously, the SOFC dynamic system meets the following two assumptions.

Assumption 1: The partial derivative of \( f(\cdot) \) with respect to the control input \( u(k) \) and disturbance \( d(k) \) is continuous.

Assumption 2: The system (12) is generalised Lipschitz to \( u \) and \( d \), i.e. satisfying \( \Delta_{u}(k + 1) \leq C_{1} | \Delta_{u}(k) | \) and \( \Delta_{d}(k + 1) \leq C_{2} | \Delta_{d}(k) | \) for \( \forall k, | \Delta_{u}(k) | \neq 0 \) and \( | \Delta_{d}(k) | \neq 0 \), where \( \Delta_{u}(k) = y(k + 1) - y(k), \Delta_{u}(k) = u(k) - u(k - 1), \) and \( \Delta_{d}(k) = d(k) - d(k - 1) \), and \( C_{1}, C_{2} \) are constants.

Remark 2: Assumption 1 is a typical condition of control system design for general non-linear systems. Assumption 2 limits the rates of changes of the system outputs driven by the changes of the control inputs.

### 3 Main results

In this section, we will propose an improved model free adaptive constrained control approach using the improved constrained CFDL modelling method. Main contributions in the following works include as: (i) proposed an improved CFDL data-driven modelling method; (ii) proposed an unknown pseudo-partial derivative estimation algorithm; and (iii) proposed a data-driven constrained control algorithm via anti-windup scheme.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Representation</th>
</tr>
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<tbody>
<tr>
<td>( T_{s} )</td>
<td>1273</td>
<td>K</td>
<td>absolute temperature</td>
</tr>
<tr>
<td>( F_{p} )</td>
<td>96,485</td>
<td>J/mol K</td>
<td>Faraday's constant</td>
</tr>
<tr>
<td>( R_{g} )</td>
<td>8.314</td>
<td>J/mol</td>
<td>universal gas constant</td>
</tr>
<tr>
<td>( E_{0} )</td>
<td>1.18</td>
<td>V</td>
<td>ideal standard potential</td>
</tr>
<tr>
<td>( N_{0} )</td>
<td>384</td>
<td>–</td>
<td>number of cells in series in the stack</td>
</tr>
<tr>
<td>( K_{v} )</td>
<td>( 0.996 \times 10^{-3} )</td>
<td>mol/(s A)</td>
<td>constant, ( K = N_{0}/4F_{p} )</td>
</tr>
<tr>
<td>( K_{v0} )</td>
<td>( 8.32 \times 10^{-4} )</td>
<td>mol/(s Pa)</td>
<td>valve molar constant for hydrogen</td>
</tr>
<tr>
<td>( K_{v0O} )</td>
<td>( 2.77 \times 10^{-5} )</td>
<td>mol/(s Pa)</td>
<td>valve molar constant for water</td>
</tr>
<tr>
<td>( K_{D2} )</td>
<td>( 2.49 \times 10^{-5} )</td>
<td>mol/(s Pa)</td>
<td>valve molar constant for oxygen</td>
</tr>
<tr>
<td>( \tau_{H2} )</td>
<td>26.1</td>
<td>s</td>
<td>response time of hydrogen flow</td>
</tr>
<tr>
<td>( \tau_{D2} )</td>
<td>78.3</td>
<td>s</td>
<td>response time of water flow</td>
</tr>
<tr>
<td>( \tau_{O2} )</td>
<td>2.91</td>
<td>s</td>
<td>response time of oxygen flow</td>
</tr>
<tr>
<td>( \tau_{H2O} )</td>
<td>1.145</td>
<td>–</td>
<td>ratio of hydrogen to oxygen</td>
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<tr>
<td>( r )</td>
<td>0.126</td>
<td>Ω</td>
<td>Ohmic loss</td>
</tr>
<tr>
<td>( \tau_{O} )</td>
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<td>–</td>
<td>time constant of the fuel processor</td>
</tr>
<tr>
<td>( \Delta_{1} )</td>
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<td>–</td>
<td>Tafel constant</td>
</tr>
<tr>
<td>( \Delta_{2} )</td>
<td>0.11</td>
<td>–</td>
<td>Tafel slope</td>
</tr>
<tr>
<td>( \Delta_{L} )</td>
<td>800</td>
<td>A</td>
<td>limiting current density</td>
</tr>
</tbody>
</table>

Fig. 3 Voltage–current characteristics of an open-loop SOFC
3.1 Improved CFDL data driven modelling

Data driven dynamic linearisation techniques have been proposed by Hou in [23–25], including CFDL, PFDL, and FFDL. However, the disturbance problem is not really considered in these dynamic linearisation techniques. Here, we give an improved CFDL method.

**Theorem 1:** For the non-linear system (12) under Assumptions 1 and 2 holding, there must exist parameters \( \phi(k), \psi(k) \), called PFDL, system (12) can be transformed into the following CFDL description when \( |\Delta u(k)| \neq 0 \)

\[
\dot{y}(k + 1) = \phi(k)\Delta u(k) + \psi(k)\Delta d(k) + \xi(k)
\]

(13)

where \( \phi(k) \leq C_1, |\psi(k)| \leq C_2 \).

**Proof:** According to Assumptions 1 and 2, and mean value theorem, non-linear system (12) can be easily described as the following

\[
\dot{y}(k + 1) = \frac{\partial f^*}{\partial u(k)} u(k) + \frac{\partial f^*}{\partial d(k)} d(k) + \xi(k)
\]

(14)

where \( \frac{\partial f^*}{\partial u(k)} \) represents the partial derivative value of \( f(\cdot) \) at \( u(k) \), \( \frac{\partial f^*}{\partial d(k)} \) represents the partial derivative value of \( f(\cdot) \) at \( d(k) \), and

\[
\xi(k) = f\left(y(k), \ldots, y(k-n_k), u(k-1), \ldots, u(k-n_o), d(k-1), \ldots, d(k-n_d)\right) - f\left(y(k-1), \ldots, y(k-n_y), u(k-1), \ldots, u(k-n_u), d(k-1), \ldots, d(k-n_d-1)\right)
\]

(15)

Since \( |\Delta u(k)| \neq 0 \), for each fixed \( k \), we consider the following equation with

\[
\xi(k) = \eta(k)|\Delta u(k)|
\]

(16)

Hence, (15) can be represented as

\[
\dot{y}(k + 1) = \phi(k)\Delta u(k) + \psi(k)\Delta d(k)
\]

(17)

where

\[
\phi(k) = \frac{\partial f^*}{\partial u(k)} + \eta(k)
\]

\[
\psi(k) = \frac{\partial f^*}{\partial d(k)}
\]

Results \( \phi(k) \leq C_1, |\psi(k)| \leq C_2 \) are the direct conclusions of Assumption 2. \( \square \)

**Remark 3:** In (17), \( \phi(k) \) is the sensitivity of SOFC output voltage \( V_{dc} \) to control input \( q_i \), \( \psi(k) \) is the sensitivity of SOFC output voltage \( V_{dc} \) to control load \( I \).

3.2 Parameter estimation algorithm

The CFDL modelling (13) is redescribed as

\[
y(k + 1) = y(k) + \Phi^T(k)\hat{\theta}(k)
\]

(18)

where \( \hat{\theta}(k) = [\phi(k), \psi(k)]^T \), \( \Phi(k) = [\Delta u(k), \Delta d(k)]^T \). The proposed parameter identification observer has the following structure

\[
\hat{y}(k + 1) = \hat{y}(k) + \Phi^T(k)\hat{\theta}(k) + Ke_o(k)
\]

(19)

where \( e_o(k) = y(k) - \hat{y}(k) \) is the output estimation error, \( \hat{\theta}(k) = [\phi(k), \psi(k)]^T \), and the gain \( K \) is chosen as making \( F = 1 - K \) in the unit circle.

Hence, in view of (18) and (19), the output estimation error dynamics is given by

\[
e_o(k + 1) = \Phi^T(k)\hat{\theta}(k) - Fe_o(k)
\]

(20)

where \( \hat{\theta}(k) = \theta(k) - \hat{\theta}(k) \) represents the parameter estimation error. The adaptive update law for the estimated parameters \( \hat{\theta}(k) \) can be chosen as

\[
\hat{\theta}(k + 1) = \hat{\theta}(k) + \Phi(k)\Gamma(k)(e_o(k + 1) + Fe_o(k))
\]

(21)

The gain \( \Gamma(k) \) is chosen as follows

\[
\Gamma(k) = 2\left(\|\Phi(k)\|^2 + \mu\right)^{-1}
\]

(22)

where \( \mu \) is a positive constant, hence, \( \Gamma(k) \) is positive definite for all \( k \). Notice that, by virtue of assumption \( \|\Phi(k)\| \leq \gamma \), \( \Gamma(k) \) can be lower bounded as

\[
\|\Gamma(k)\| \geq \frac{2}{\gamma^2 + \mu} = \gamma > 0
\]

By taking into account (20) and (21), the estimation error dynamics can be written as

\[
e_o(k + 1) = \Phi^T(k)\hat{\theta}(k) - Fe_o(k)
\]

\[
\hat{\theta}(k + 1) = H\hat{\theta}(k)
\]

(23)

where \( H \) is given by

\[
H = I_N - \Phi(k)\Gamma(k)\Phi^T(k)
\]

and \( I \) denotes the \( (2 \times 2) \) identity matrix.

**Theorem 2:** The equilibrium \( [e_o, \hat{\theta}]^T = [0, 0] \) of the system (23) is globally uniformly stable. Furthermore, the estimation error \( e_o(k) \) converges asymptotically to 0.

**Proof:** Consider the Lyapunov function

\[
V_1(k) = Pe_o^2(k) + \lambda\hat{\theta}^T(k)\hat{\theta}(k)
\]

(24)

where \( \lambda, P \) are positive constants and \( P \) is the solution by \( P - PF^2P = Q \) with \( Q \) is positive constant. By taking into (23), we have

\[
\Delta V_1(k) = V_1(k + 1) - V_1(k)
\]

\[
= P\Phi^T(k)\hat{\theta}(k)\Phi(k) - 2PF\Phi^T(k)\hat{\theta}(k)e_o(k)
\]

\[
+ PF^2e_o^2(k) + \hat{\theta}^T(k)\left(\lambda H^T H - \lambda\right)\hat{\theta}(k) + P\hat{\theta}^2(k)
\]

\[
= -Qe_o^2(k) - \Theta^T(k)\left[\lambda\mu\Gamma^T(k)\Gamma(k) - P\right]\Theta(k)
\]

\[
- 2PFe_o(k)\Theta(k)
\]

\[
\leq -Qe_o^2(k) - \left[\lambda\mu\Gamma^T(k)\Gamma(k) - P\right]\Theta^2(k)
\]

\[
+ 2PF\Theta^2(k)\|\Theta(k)\|^2
\]

\[
\leq -c_1e_o^2(k) - c_2\Theta^2(k)
\]

where \( \Theta(k) = \Phi^T(k)\hat{\theta}(k) \), \( c_1 = Q - (1/\varsigma) \) and \( c_2 = \mu\lambda\gamma^2 - P - \varsigma P^2\gamma^2 \). Hence, \( \Delta V_1(k) \leq 0 \) provides that \( \varsigma, Q \) and \( \lambda \) satisfy the following inequalities

\[
Q > \frac{1}{\varsigma}, \quad \mu\lambda\gamma^2 - P - \varsigma P^2\gamma^2 > 0
\]

Notice that \( \Delta V_1(k) \) is negative definite in the variables \( e_o(k), \Theta(k) \). Since \( V_1(k) \) in a decreasing and non-negative function, it converges
to a constant value \( y^*_{\infty} \geq 0 \), as \( k \to \infty \), hence, \( \Delta V_1(k) \to 0 \). This implies that both \( e_o(k) \) and \( \hat{\theta}(k) \) remain bounded for all \( k \), and \( \lim_{k \to \infty} e_o(k) = 0 \). □

Remark 4: In order to make the parameter estimation law (21) has a stronger capability in tracking time-varying parameter, a reset algorithm which in [23] should be considered as following

\[
\hat{\phi}(k) = \hat{\phi}(1) , \text{ if } |\hat{\phi}(k)| \leq \epsilon \quad \text{ or } \quad \text{sign}(\hat{\phi}(k)) \neq \text{sign}(\hat{\phi}(1)) \quad (25)
\]

where \( \epsilon \) is a small positive constant and \( \hat{\phi}(1) \) is the initial value of \( \hat{\phi}(k) \).

3.3 Solution of constrained control and stability analysis

Based on the observer (19), the model free adaptive unconstrained control law is calculated as

\[
u_0(k) = u(k - 1) + \phi(k) (y^*(k + 1) - \hat{y}(k) - Ke_o(k) - \hat{\phi}(k)\Delta d(k)) \quad (26)
\]

where \( y^*(k) \) is the reference trajectory. \( \alpha \) is a given small finite positive number. Consider the input constraints (11), then adaptive constrained controller is described as

\[
u(t) = \text{Sat} (u(k - 1) + \text{Sat} (\{u_0(k) - u(k - 1)\}), T\hat{y}_{\min}, T\hat{y}_{\max} \}, \hat{y}_{\min}, \hat{y}_{\max}) \quad (27)
\]

where \( T \) is the sampling time and Sat(·) function is defined as

\[
\text{Sat}(a, b, c) = \begin{cases} b & a \leq b \\ a & b < a < c \\ c & a \geq c \end{cases}
\]

Due to the dynamic constraints in the close-loop SOFC control, a anti-windup compensator is designed to accommodate the reference trajectory \( y^*(k) \). The compensation signal \( \zeta(k) \) is designed as following

\[
\zeta(k + 1) = \sigma \zeta(k) + \hat{\phi}(u_0(k) - u(k)) \quad (28)
\]

where \( \sigma \) lies in the unit disk. Hence, the controller (26) becomes as

\[
u_0(k) = u(k - 1) + \phi(k) (y^*(k + 1) - \hat{y}(k) - \sigma \zeta(k)) - Ke_o(k) - \hat{\phi}(k)\Delta d(k)) \quad (29)
\]

Defining observer tracking error \( e(k) = y^*(k) - \hat{y}(k) - \zeta(k) \) under dynamic constraints (11), and combining (28), thus

\[
e(k + 1) = y^*(k + 1) - \hat{y}(k + 1) - \zeta(k + 1)
\]

\[
= y^*(k + 1) - \hat{y}(k) - \hat{\phi}(k)u_0(k) - \hat{\phi}(k)\Delta d(k)
\]

\[
- Ke_o(k) - \sigma \zeta(k) - \hat{\phi}(k)(u_0(k) - u(k))
\]

\[
= y^*(k + 1) - \hat{y}(k) - \sigma \zeta(k) - \hat{\phi}(k)\Delta d(k) - Ke_o(k)
\]

Substituting (29) into (30) gives

\[
e(k + 1) = \frac{\alpha}{\phi_i^2(k) + \alpha} (y^*(k + 1) - \hat{y}(k)
\]

\[
- \sigma \zeta(k) - Ke_o(k) - \hat{\phi}(k)\Delta d(k))
\]

The convergence and tracking performance analysis for data-driven model free adaptive constrained control law (27) and (29) are given in Theorem 3.

Theorem 3: For given \(|y^*(k) - y^*(k - 1)| \leq \Delta y^* \), using the model free adaptive constrained control law (27) and (29), the solution of close-loop observer error system (31) is uniformly ultimately bounded (UUB) [30] for all \( k \) with ultimate bound 

\[
limit_{k \to \infty} |e(k)| \leq a_2/(1 - a_1)
\]

where \( \Delta y^* \) is a given positive constant

\[
a_1 = \frac{\alpha}{\phi_i^2(k) + \alpha}
\]

\[
a_2 = \frac{\alpha}{\phi_i^2(k) + \alpha} |\Delta y^* + (1 - \sigma)\zeta(k) - Ke_o(k) - \hat{\phi}(k)\Delta d(k)|
\]

Proof: Taking the absolute value of (31) becomes

\[
|e(k + 1)| = \frac{\alpha}{\phi_i^2(k) + \alpha} (y^*(k + 1) - \hat{y}(k) - \sigma \zeta(k)
\]

\[
- Ke_o(k) - \hat{\phi}(k)\Delta d(k))
\]

\[
= \frac{\alpha}{\phi_i^2(k) + \alpha} (y^*(k + 1) - y^*(k) + e(k) + (1 - \sigma)\zeta(k)
\]

\[
- Ke_o(k) - \hat{\phi}(k)\Delta d(k)
\]

\[
\leq \frac{\alpha}{\phi_i^2(k) + \alpha} |e(k)| + \frac{\alpha}{\phi_i^2(k) + \alpha} |\Delta y^* + (1 - \sigma)\zeta(k) - Ke_o(k) - \hat{\phi}(k)\Delta d(k)|
\]

\[
= a_1 |e(k)| + a_2
\]

Choosing a Lyapunov function as \( V_2(k) = |e(k)| \), from (32), one has

\[
\Delta V_2(k + 1) = |e(k + 1)| - |e(k)| = (1 - a_1)V_2(k) + a_2
\]

Since \( 0 \leq a_1 < 1 \) and \( a_2 \) is bounded, according to the lemma in [30], using the constrained control law (27) and (29), the results of close-loop observer system (31) are UUB for all \( k \) with ultimate bound 

\[
limit_{k \to \infty} |\hat{e}(k)| \leq a_2/(1 - a_1)
\]

Corollary 1: Under the controller (27) and (29), together with the observer (19), adaptive laws (21), we can guarantee that the system (12) tracking error \( \hat{e}(k) = y^*(k) - \hat{y}(k) \) is UUB with ultimate bound 

\[
limit_{k \to \infty} |\hat{e}(k)| \leq a_2/(1 - a_1)
\]

Proof: Since

\[
\hat{e}(k) = e_o(k) - e(k)
\]

Taking the absolute value and limiting on both sides of (33), we obtain

\[
\lim_{k \to \infty} |\hat{e}(k)| \leq \lim_{k \to \infty} |e_0(k)| + \lim_{k \to \infty} |e(k)| \leq \frac{a_2}{1 - a_1}
\]

So the tracking error \( \hat{e}(k) \) is UUB for all \( k \) with ultimate bound 

\[
limit_{k \to \infty} |\hat{e}(k)| \leq a_2/(1 - a_1)
\]

Remark 5: The controller (29) can be subdivided into two parts: feedback control and feedforward control, which is described as

\[
u_0(k) = u_0(k) + u_02(k)
\]

where

Feedback control:

\[
u_01(k) = u(k - 1) + \frac{\hat{\phi}(k)}{\phi_i^2(k) + \alpha} (y^*(k + 1) - \hat{y}(k) - \sigma \zeta(k) - Ke_o(k))
\]

Feedforward control: \( u_02(k) = -\frac{\hat{\phi}(k)\hat{\phi}(k)\Delta d(k)}{\phi_i^2(k) + \alpha} \)

The feedforward control loop is used to keep the output voltage steady under drastic measurable current load \( I \) changes.
Remark 6: In MFAC [23], consider the following control input criterion function

\[ J(u(k)) = |y^*(k+1) - y(k+1)|^2 + \varrho |u(k) - u(k-1)|^2 \]

Although, weight value \( \varrho \) of performance index function is used to deal with the control input saturation problems. There are three main problems in performance index function method, they are: (i) weight value \( \varrho \) is time-varying in theory and (ii) the relationship between control input saturations and weight value \( \varrho \) is very difficult to obtain. In [26], the proposed control method can deal with the rate saturation problem, but the magnitude saturation cannot be considered. In this paper, a novel dynamic anti-windup is first proposed for magnitude and rate saturations in control system design. The proposed constrained control approach in this paper is effective to avoid the above three difficulties of [23, 26].

Remark 7: As we know, optimal control is core method in MPC, which is used to solve the control input constraints. In this paper, we adopt different idea to deal with control input constraints, that is, compensation signal \( \zeta(k) \) be used for regulation reference input \( y^*(k) \) to ensure that control input \( u(k) \) is working within the constraints.

To give a clear idea of the overall proposed SOFC control system design procedure, we give a flowchart as shown in Fig. 4.

4 Simulation results

In this section, the proposed improved data driven model free adaptive constrained control algorithm discussed above is applied to the

Fig. 4 Flowchart of the proposed SOFC control system design procedure

Fig. 5 Simulation results with the proposed control, MFAC and adaptive NNC
SOFC control system to achieve safe fuel utilisation and operating constraints when the current load $I$ and voltage output $V_{dc}$ are online measurable.

As an independent power source candidate, we must ensure that output voltage of the SOFC system is expected to be at a desired constant value. The external current load $I$ will directly affect the output voltage of the SOFC system. In normal working conditions, the current load $I$ of the SOFC system is 300 A. The reference of the output voltage as

$$V_{dc}^* = \begin{cases} 
332.8 \text{ V} & t \leq 1000 \\
350 \text{ V} & 1000 < t \leq 2000 \\
332.8 \text{ V} & t > 2000 
\end{cases}$$

In simulations, assuming that current load $I$ (A) changes as shown in Fig. 5.

---

**Fig. 6** Simulation results with the proposed control

- a Fuel utilisation $\rho$
- b Compensation signal $\zeta(k)$

**Fig. 7** PPD parameters estimation
The safe fuel utilisation and maintain operational constraints are considered as \( [\rho_{\text{min}}, \rho_{\text{max}}] = [0.7, 0.9] \) and \( [q_{\text{min}}, q_{\text{max}}] = [0, 1.2]\)mol/s, and \( [\dot{q}_{\text{min}}, \dot{q}_{\text{max}}] = [-0.7, 0.7]\)mol/s\(^2\), respectively. For the proposed control law, we choose the sampling time \( T = 1s \). The parameters of proposed model free adaptive constrained control law in Section 3 are \( \mu = 10 \), \( K = 0.99 \), \( \sigma = 0.25 \). The initial values of observer parameters \( \theta(0) = [\phi(0), \psi(0)]^T = [0.055, 0]^T \).

A typical system response using the proposed algorithm, MFAC \( ^{[23]} \), adaptive NN (ANN) \( ^{[31]} \) are depicted for large current load step changes, as shown in Fig. 5, in which we can see that fairly good tracking performance is obtained with proposed constrained control. From Fig. 5, we observe that the tracking error of proposed control method is faster to a small neighbourhood of zero than MFAC and ANN. The fuel utilisation \( \rho \) and the anti-windup compensation signal \( \zeta(k) \) are shown in Fig. 6. From Fig. 6, it can be seen that the proposed control method can keep fuel utilisation \( \rho \) within a safe range 0.7–0.9 as long as possible. In the whole tracking trajectory, tuning PPD parameters are shown in Fig. 7.

5 Conclusion

A model free adaptive constrained control strategy has been developed based on proposed improved CFDL data driven modelling for the SOFC system, which provides an alternative to the SOFC control problem. A dynamic constraints unit with anti-windup scheme is adopted to keep fuel utilisation within a safe range as long as possible. The proposed control law has the real-time implementation advantage of not requiring any iterative computation for determining the control input and it is particularly effective while the explicit analytical model of non-linear systems is difficult to develop. The simulation results have validated the proposed data driven model free adaptive constrained non-linear control algorithm.

6 Acknowledgments

The authors sincerely thank the editor and all the anonymous reviewers for their valuable comments which helped them improve the manuscript of this paper. This work was supported by National Natural Science Foundation of China (61503156), the Fundamental Research Funds for the Central Universities (JUSRP11562, NJ20150011).

7 References